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Excited scalar mesons in a chiral quark model

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Abstract

First radial excitations of the isoscalar and isovector scalar mesons $f_0(400-1200)$, $f_0(980)$ and $a_0(980)$ are investigated in the framework of a nonlocal version of a chiral quark model of the Nambu–Jona-Lasinio (NJL) type. It is shown that $f_0(1370)$, $f_J(1710)$ and $a_0(1450)$ are the first radially excited states of $f_0(400-1200)$, $f_0(980)$ and $a_0(980)$ which are ground states of the scalar meson nonet. The mesons' masses and strong decay widths are calculated. The scalar resonance $f_0(1500)$ is supposed to be a glueball. The status of $K_0^*(1430)$ is discussed.

Keywords: quark model, chiral symmetry, scalar mesons, radial excitations

1. Introduction

A correct description of both the ground and excited states of scalar mesons encounters a variety of complex problems. Let us point out some of them. i) For a long time, the experimental status of the lightest scalar isoscalar singlet meson was unclear. In some papers, the resonance $f_0(1370)$ was considered as that particle [1], and it was not until 1998 that the resonance $f_0(400-1200)$ was included into the summary tables of PDG review[1] [2]. ii) The scalar isoscalar states, as having quantum numbers of vacuum, are most probably get mixed with glueballs [3]. iii) There is also a lot of problems related to the description of $f_0(980)$ and $a_0(980)$. Their unusual experimental branching ratios for several decays have brought forth different ideas concerning the structure of the mesons. Among them, there are the quark-antiquark model [1, 3, 4], the four-quark model [5] and the kaon molecule model [6]. iv) The strange meson $K_0^*(1430)$ seems too heavy to be the ground state: 1 GeV is more characteristic of the ground meson states (see [7]).

The description of the ground and excited states of the pion, kaon and the vector meson nonet in the framework of a nonlocal version of the NJL model has been done in our earlier papers [8, 9, 10]. Here we intend to study the ground and first radially excited states of η, η' mesons and of the scalar meson nonet.

To produce correct masses for the ground states of η and η', we, as usual, introduce 't Hooft interaction [11, 12]. Although our model is nonlocal, which is reflected in the presence of form factors in the four-quark vertices, we, nevertheless, assume the 't Hooft term local. The form factors in scalar channels of quark current-current interaction are chosen identical to those in the pseudoscalar channel. This is a requirement of the global chiral symmetry of quark interaction. With that assumption, there is no need for additional parameters in the form factors of scalar quark vertices. Therefore, the masses of scalar mesons can be immediately predicted after fixing the form factor parameters by the pseudoscalar meson masses from experiment. As a result, we have found that the model masses of the radial excitations of scalar isoscalar mesons are close to the experimentally observed $f_0(1370)$, $f_J(1710)$[2], $a_0(1450)$ mesons. This allows us to interpret them as the first radial excitations of mesons $f_0(400-1200)$, $f_0(980)$ and $a_0(980)$. As to the state $f_0(1500)$, we are inclined to consider it as a glueball. In our further works we will take into account possible mixing of the $\bar{q}q$ scalar meson states with glueballs [3].

Concerning the strange scalar K_0^*, we think that the state with mass 1430 MeV is much likely a radial excitation of a light and wide resonance with mass about 960 MeV (see [7]). Further discussion on this problem is given in conclusion.

As to the ground states $a_0(980)$ and $f_0(980)$, the detailed discussion on their internal structure and properties is beyond the scope of our paper.

Our paper is organized as follows. In Sec. 2, we introduce the chiral quark

[1] However, in earlier editions of PDG the light σ state still could be found; it was excluded later.

[2] We assume hereafter $f_J(1710)$ is an isoscalar ($J = 0$).

Lagrangian with nonlocal four-quark vertices and local 't Hooft interaction. In Sec. 3, we calculate the effective Lagrangian for isovector and strange mesons in the one-loop approximation. There we renormalize meson fields and transform the free part of the Lagrangian to the diagonal form and obtain meson mass formulae. Section 4 is devoted to isoscalar mesons where we find masses and mixing coefficients. The model parameters are discussed in Sec. 5. In Sec. 6, we calculate the widths of major strong decays of excited states of a_0, σ and f_0 mesons. The results of our work and possible ways to improve the model are discussed in Sec. 7. Some details of the calculations fulfilled in Sec. 4 and 6 are given in Appendices A and B.

2. $U(3) \times U(3)$ chiral Lagrangian with excited meson states and 't Hooft interaction

We use a nonlocal separable four-quark interaction of a current-current form which admits nonlocal vertices (form factors) in the quark currents, and a pure local six-quark 't Hooft interaction [11, 12]:

$$\mathcal{L}(\bar{q}, q) = \int d^4x\, \bar{q}(x)(i\partial\!\!\!/ - m^0)q(x) + \mathcal{L}^{(4)}_{\mathrm{int}} + \mathcal{L}^{(6)}_{\mathrm{int}}, \tag{1}$$

$$\mathcal{L}^{(4)}_{\mathrm{int}} = \int d^4x \sum_{a=0}^{8}\sum_{i=1}^{N} \frac{G}{2}[j^a_{S,i}(x)j^a_{S,i}(x) + j^a_{P,i}(x)j^a_{P,i}(x)], \tag{2}$$

$$\mathcal{L}^{(6)}_{\mathrm{int}} = -K\left[\det\left[\bar{q}(1+\gamma_5)q\right] + \det\left[\bar{q}(1-\gamma_5)q\right]\right]. \tag{3}$$

Here, m^0 is the current quark mass matrix ($m^0_u \approx m^0_d$) and $j^a_{S(P),i}$ denotes the scalar (pseudoscalar) quark currents

$$j^a_{S(P),i}(x) = \int d^4x_1 d^4x_2\, \bar{q}(x_1)F^a_{S(P),i}(x;x_1,x_2)q(x_2) \tag{4}$$

where $F^a_{S(P),i}(x;x_1,x_2)$ are the scalar (pseudoscalar) nonlocal quark vertices. To describe the first radial excitations of mesons, we take the form factors in momentum space as follows (see [8, 9, 10]),

$$F^a_{S,j}(\mathbf{k}) = \lambda^a f^a_j, \qquad F^a_{P,j} = i\gamma_5\lambda^a f^a_j \tag{5}$$

$$f^a_1 \equiv 1, \quad f^a_2 \equiv f_a(\mathbf{k}) = c_a(1 + d_a\mathbf{k}^2), \tag{6}$$

where λ^a are Gell–Mann matrices, $\lambda^0 = \sqrt{\frac{2}{3}}\mathbf{1}$, with $\mathbf{1}$ being the unit matrix. Here, we consider the form factors in the rest frame of mesons [3].

[3] The form factors depend on the transversal parts of the relative momentum of quark-antiquark pairs $k_\perp = k - \frac{k\cdot P}{P^2}P$, where k and P are the relative and total momenta of a quark-antiquark pair, respectively. Then, in the rest frame of mesons, $\mathbf{P}_{meson} = 0$, the transversal momentum is $k_\perp = (0, \vec{k})$, and we can define the form factors as depending on the 3-dimensional momentum $\vec{k}$ alone.

The part of the Lagrangian (1), describing the ground states and first radial excitations, can be rewritten in the following form (see [11] and [12]):

$$\begin{aligned}
\mathcal{L} &= \int d^4x \Big\{ \bar{q}(x)(i\partial\!\!\!/ - m^0)q(x) + \frac{G}{2}\sum_{a=0}^{8}\left[\left(j^a_{S,2}\right)^2 + \left(j^a_{P,2}\right)^2\right] + \\
&\frac{1}{2}\sum_{a=1}^{9}\left[G_a^{(-)}\left(\bar{q}(x)\tau_a q(x)\right)^2 + G_a^{(+)}\left(\bar{q}(x)i\gamma_5\tau_a q(x)\right)^2\right] + \\
&G_{us}^{(-)}(\bar{q}(x)\lambda_u q(x))(\bar{q}(x)\lambda_s q(x)) + G_{us}^{(+)}(\bar{q}(x)i\gamma_5\lambda_u q(x))(\bar{q}(x)i\gamma_5\lambda_s q(x))\Big\},
\end{aligned} \tag{7}$$

where

$$\begin{aligned}
&\tau_i = \lambda_i \quad (i = 1, ..., 7), \quad \tau_8 = \lambda_u = (\sqrt{2}\lambda_0 + \lambda_8)/\sqrt{3}, \\
&\tau_9 = \lambda_s = (-\lambda_0 + \sqrt{2}\lambda_8)/\sqrt{3}, \\
&G_1^{(\pm)} = G_2^{(\pm)} = G_3^{(\pm)} = G \pm 4Km_s I_1(m_s), \\
&G_4^{(\pm)} = G_5^{(\pm)} = G_6^{(\pm)} = G_7^{(\pm)} = G \pm 4Km_u I_1(m_u), \\
&G_u^{(\pm)} = G \mp 4Km_s I_1(m_s), \quad G_s^{(\pm)} = G, \quad G_{us}^{(\pm)} = \pm 4\sqrt{2}Km_u I_1(m_u).
\end{aligned} \tag{8}$$

Here m_u and m_s are the constituent quark masses and $I_1(m_q)$ is the integral which for an arbitrary n is defined as follows

$$I_n(m_q) = \frac{-iN_c}{(2\pi)^4}\int_{\Lambda_3} d^4k \frac{1}{(m_q^2 - k^2)^n}. \tag{9}$$

The 3-dimensional cut-off Λ_3 in (9) is implemented to regularize the divergent integrals[4].

3. The masses of isovector and strange mesons (ground and excited states)

After bosonization, the part of Lagrangian (7), describing the isovector and strange mesons, takes the form

$$\begin{aligned}
&\mathcal{L}(a_{0,1}, K^*_{0,1}, \pi_1, K_1, a_{0,2}, K^*_{0,2}, \pi_2, K_2) = -\frac{a_{0,1}^2}{2G_{a_0}} - \frac{{K^*_{0,1}}^2}{G_{K_0^*}} - \frac{\pi_1^2}{2G_\pi} - \frac{K_1^2}{G_K} - \\
&\frac{1}{2G}(a_{0,2}^2 + 2(K^*_{0,2})^2 + \pi_2^2 + 2K_2^2) - \\
&iN_c \mathrm{Tr}\ln\left[1 + \frac{1}{i\partial\!\!\!/ - m}\sum_{a=1}^{7}\sum_{j=1}^{2}\lambda_a\left[\sigma_j^a + i\gamma_5\varphi_j^a\right]f_j^a\right]
\end{aligned} \tag{10}$$

[4]For instance, $I_1(m) = \frac{N_c m^2}{8\pi^2}\left[x\sqrt{x^2+1} - \ln(x + \sqrt{x^2+1})\right]\big|_{x=\Lambda_3/m}$.

where $m = \mathrm{diag}(m_u, m_d, m_s)$ is the matrix of constituent quark masses ($m_u \approx m_d$), σ_j^a and ϕ_j^a are the scalar and pseudoscalar fields: $\sum_{a=1}^3 (\sigma_j^a)^2 \equiv a_{0,j}^2 = (a_{0,j}^0)^2 + 2a_{0,j}^+ a_{0,j}^-$, $\sum_{a=4}^7 (\sigma_j^a)^2 \equiv 2{K_{0,j}^*}^2 = 2(\bar{K}_{0,j}^*)^0 (K_{0,j}^*)^0 + 2(\bar{K}_{0,j}^*)^+ (K_{0,j}^*)^-$, $\sum_{a=1}^3 (\varphi_j^a)^2 \equiv \pi_j^2 = (\pi_j^0)^2 + 2\pi_j^+ \pi_j^-$, $\sum_{a=4}^7 (\varphi_j^a)^2 \equiv 2K_j^2 = 2\bar{K}_j^0 K_j^0 + 2\bar{K}_j^+ K_j^-$. As to the coupling constants G_a, they will be defined later (see Sect. 5 and (8)).

The free part of Lagrangian (10) has the following form

$$\mathcal{L}^{(2)}(\sigma, \varphi) = \frac{1}{2} \sum_{i,j=1}^{2} \sum_{a=1}^{7} \left(\sigma_i^a K_{\sigma,ij}^a(P) \sigma_j^a + \varphi_i^a K_{\varphi,ij}^a(P) \varphi_j^a \right) \tag{11}$$

where the coefficients $K_{\sigma(\varphi),ij}^a(P)$ are given below,

$$K_{\sigma(\varphi),ij}^a(P) = -\delta_{ij} \left[\frac{\delta_{i1}}{G_a^{(\mp)}} + \frac{\delta_{i2}}{G} \right] - \\ iN_c \mathrm{Tr} \int_{\Lambda_3} \frac{d^4k}{(2\pi)^4} \frac{1}{\not{k} + \not{P}/2 - m_q^a} r^{\sigma(\varphi)} f_i^a \frac{1}{\not{k} - \not{P}/2 - m_{q'}^a} r^{\sigma(\varphi)} f_j^a, \tag{12}$$

$$r^\sigma = 1, \quad r^\phi = i\gamma_5, \tag{13}$$

$$m_q^a = m_u \quad (a = 1, ..., 7); \quad m_{q'}^a = m_u \quad (a = 1, ..., 3); \quad m_{q'}^a = m_s \quad (a = 4, ..., 7), \tag{14}$$

with m_u and m_s being the constituent quark masses and f_j^a defined in (6). Integral (12) is evaluated by expanding in the meson field momentum P. To order P^2, one obtains

$$\begin{aligned}
K_{\sigma(\varphi),11}^a(P) &= Z_{\sigma(\varphi),1}^a (P^2 - (m_q^a \pm m_{q'}^a)^2 - M_{\sigma^a(\varphi^a),1}^2), \\
K_{\sigma(\varphi),22}^a(P) &= Z_{\sigma(\varphi),2}^a (P^2 - (m_q^a \pm m_{q'}^a)^2 - M_{\sigma^a(\varphi^a),2}^2), \\
K_{\sigma(\varphi),12}^a(P) &= K_{\sigma(\varphi),21}^a(P) = \gamma_{\sigma(\varphi)}^a (P^2 - (m_q^a \pm m_{q'}^a)^2),
\end{aligned} \tag{15}$$

where

$$Z_{\sigma,1}^a = 4I_2^a, \qquad Z_{\sigma,2}^a = 4I_2^{ffa}, \qquad \gamma_\sigma^a = 4I_2^{fa}, \tag{16}$$

$$Z_{\varphi,1}^a = Z Z_{\sigma,1}^a, \qquad Z_{\varphi,2}^a = Z_{\sigma,2}^a, \qquad \gamma_\varphi^a = Z^{1/2} \gamma_\sigma^a \tag{17}$$

and

$$M_{\sigma^a(\varphi^a),1}^2 = (Z_{\sigma(\varphi),1}^a)^{-1} \left[\frac{1}{G_a^{(\mp)}} - 4(I_1(m_q^a) + I_1(m_{q'}^a)) \right] \tag{18}$$

$$M_{\sigma^a(\varphi^a),2}^2 = (Z_{\sigma(\varphi),2}^a)^{-1} \left[\frac{1}{G} - 4(I_1^{ffa}(m_q^a) + I_1^{ffa}(m_{q'}^a)) \right]. \tag{19}$$

The factor Z here appears due to account of $\pi - a_1$-transitions [4, 9],

$$Z = 1 - \frac{6m_u^2}{M_{a_1}^2}, \tag{20}$$

and the integrals $I_2^{f..f}$ contain form factors:

$$I_2^{f..f_a}(m_q^a, m_{q'}^a) = \frac{-iN_c}{(2\pi)^4}\int_{\Lambda_3} d^4k \frac{f_a(\mathbf{k})..f_a(\mathbf{k})}{((m_q^a)^2-k^2)((m_{q'}^a)^2-k^2)}. \qquad (21)$$

Further, we consider only the scalar isovector and strange mesons because the masses of the pseudoscalar mesons have been already described in [9].

After the renormalization of the scalar fields

$$\sigma_i^{ar} = \sqrt{Z_{\sigma,i}^a}\sigma_i^a \qquad (22)$$

the part of Lagrangian (11) which describes the scalar mesons takes the form

$$\begin{aligned}\mathcal{L}_{a_0}^{(2)} &= \frac{1}{2}\left(P^2-4m_u^2-M_{a_0,1}^2\right)a_{0,1}^2+\Gamma_{a_0}\left(P^2-4m_u^2\right)a_{0,1}a_{0,2}+\\ &\quad \frac{1}{2}\left(P^2-4m_u^2-M_{a_0,2}^2\right)a_{0,2}^2, \end{aligned}\qquad (23)$$

$$\begin{aligned}\mathcal{L}_{K_0^*}^{(2)} &= \frac{1}{2}\left(P^2-(m_u+m_s)^2-M_{K_0^*,1}^2\right)K_{0,1}^{*2}+\Gamma_{K_0^*}\left(P^2-(m_u+m_s)^2\right)K_{0,1}^*K_{0,2}^*+\\ &\quad \frac{1}{2}\left(P^2-(m_u+m_s)^2-M_{K_0^*,2}^2\right)K_{0,2}^{*2}, \end{aligned}\qquad (24)$$

where

$$\Gamma_{\sigma^a} = \frac{I_2^{f_a}}{\sqrt{I_2 I_2^{ff_a}}}. \qquad (25)$$

After the transformations of the meson fields

$$\begin{aligned}\sigma^a &= \cos(\theta_{\sigma,a}-\theta_{\sigma,a}^0)\sigma_1^{ar}-\cos(\theta_{\sigma,a}+\theta_{\sigma,a}^0)\sigma_2^{ar},\\ \hat{\sigma}^a &= \sin(\theta_{\sigma,a}-\theta_{\sigma,a}^0)\sigma_1^{ar}-\sin(\theta_{\sigma,a}+\theta_{\sigma,a}^0)\sigma_2^{ar}, \end{aligned}\qquad (26)$$

Lagrangians (23) and (24) take the diagonal form:

$$L_{a_0}^{(2)} = \tfrac{1}{2}(P^2-M_{a_0}^2)\ a_0^2+\tfrac{1}{2}(P^2-M_{\hat{a}_0}^2)\hat{a}_0^2, \qquad (27)$$

$$L_{K_0^*}^{(2)} = \tfrac{1}{2}(P^2-M_{K_0^*}^2)\ K_0^{*2}+\tfrac{1}{2}(P^2-M_{\hat{K}_0^*}^2)\hat{K}_0^{*2}. \qquad (28)$$

Here we have

$$\begin{aligned}M_{(a_0,\hat{a}_0)}^2 &= \frac{1}{2(1-\Gamma_{a_0}^2)}\Big[M_{a_0,1}^2+M_{a_0,2}^2\pm\\ &\quad \sqrt{(M_{a_0,1}^2-M_{a_0,2}^2)^2+(2M_{a_0,1}M_{a_0,2}\Gamma_{a_0})^2}\Big]+4m_u^2, \end{aligned}\qquad (29)$$

$$\begin{aligned}M_{(K_0^*,\hat{K}_0^*)}^2 &= \frac{1}{2(1-\Gamma_{K_0^*}^2)}\Big[M_{K_0^*,1}^2+M_{K_0^*,2}^2\pm\\ &\quad \sqrt{(M_{K_0^*,1}^2-M_{K_0^*,2}^2)^2+(2M_{K_0^*,1}M_{K_0^*,2}\Gamma_{K_0^*})^2}\Big]+(m_u+m_s)^2, \end{aligned}\qquad (30)$$

Table 1: The mixing coefficients for the ground and first radially excited states of the scalar and pseudoscalar isovector and strange mesons. The caret symbol marks the excited states.

	a_0	$\hat{a}_0$
$a_{0,1}$	0.87	0.82
$a_{0,2}$	0.22	-1.17

	K_0^*	$\hat{K}_0^*$
$K_{0,1}^*$	0.83	0.89
$K_{0,2}^*$	0.28	-1.11

	π	$\hat{\pi}$
π_1	1.00	0.54
π_2	0.01	-1.14

	K	$\hat{K}$
K_1	0.96	0.56
K_2	0.09	-1.11

and

$$\tan 2\bar{\theta}_{\sigma,a} = \sqrt{\frac{1}{\Gamma^2_{\sigma^a}} - 1}\,\left[\frac{M^2_{\sigma^a,1} - M^2_{\sigma^a,2}}{M^2_{\sigma^a,1} + M^2_{\sigma^a,2}}\right], \qquad 2\theta_{\sigma,a} = 2\bar{\theta}_{\sigma,a} + \pi, \tag{31}$$

$$\sin\theta^0_{\sigma,a} = \sqrt{\frac{1+\Gamma_{\sigma^a}}{2}}. \tag{32}$$

The caret symbol stands for the first radial excitations of mesons. Transformations (26) express the "physical" fields σ and $\hat{\sigma}$ through the "bare" ones σ_i^{ar} and for calculations, these equations must be inverted. For practical use, we collect the values of the inverted equations for the scalar and pseudoscalar fields[5] in Table 1.

4. The masses of isoscalar mesons (the ground and excited states)

The 't Hooft interaction effectively gives rise to the additional four-quark vertices in the isoscalar part of Lagrangian (7):

$$\mathcal{L}_{\text{isosc}} = \sum_{a,b=8}^{9} \left[(\bar{q}\tau_a q) T^S_{ab} (\bar{q}\tau_b q) + (\bar{q} i\gamma_5 \tau_a q) T^P_{ab} (\bar{q} i\gamma_5 \tau_b q)\right] \tag{33}$$

where $T^{S(P)}$ is a matrix with elements defined as follows (for the definition of $G_u^{(\mp)}$, $G_s^{(\mp)}$ and $G_{us}^{(\mp)}$ see (8))

$$\begin{array}{ll} T^{S(P)}_{88} = G_u^{(\mp)}/2, & T^{S(P)}_{89} = G_{us}^{(\mp)}/2, \\ T^{S(P)}_{98} = G_{us}^{(\mp)}/2, & T^{S(P)}_{99} = G_s^{(\mp)}/2. \end{array} \tag{34}$$

[5] Although the formulae for the pseudoscalars are not displayed here (they have been already obtained in [9]) we need the values because we are going to calculate the decay widths of processes where pions and kaons are secondary particles.

This leads to nondiagonal terms in the free part of the effective Lagrangian for isoscalar scalar and pseudoscalar mesons after bosonization

$$\mathcal{L}_{\rm isosc}(\sigma,\varphi) = -\frac{1}{4}\sum_{a,b=8}^{9}\left[\sigma_1^a (T^S)^{-1}_{ab}\sigma_1^b + \varphi_1^a (T^P)^{-1}_{ab}\varphi_1^b\right] -$$
$$\frac{1}{2G}\sum_{a=8}^{9}\left[(\sigma_2^a)^2 + (\varphi_2^a)^2\right] -$$
$$i\ {\rm Tr}\ln\left\{1 + \frac{1}{i\partial\!\!\!/ - m}\sum_{a=8}^{9}\sum_{j=1}^{2}\tau^a[\sigma_j^a + i\gamma_5\varphi_j^a] f_j^a\right\}, \qquad (35)$$

where $(T^{S(P)})^{-1}$ is the inverse of $T^{S(P)}$:

$$\begin{array}{ll} (T^{S(P)})^{-1}_{88} = 2G_s^{(\mp)}/D^{(\mp)}, & (T^{S(P)})^{-1}_{89} = (T^{S(P)})^{-1}_{98} = -2G_{us}^{(\mp)}/D^{(\mp)}, \\ (T^{S(P)})^{-1}_{99} = 2G_u^{(\mp)}/D^{(\mp)}, & D^{(\mp)} = G_u^{(\mp)}G_s^{(\mp)} - (G_{us}^{(\mp)})^2. \end{array} \qquad (36)$$

From (35), in the one-loop approximation, one obtains the free part of the effective Lagrangian

$$\mathcal{L}^{(2)}(\sigma,\phi) = \frac{1}{2}\sum_{i,j=1}^{2}\sum_{a,b=8}^{9}\left(\sigma_i^a K^{[a,b]}_{\sigma,ij}(P)\sigma_j^b + \varphi_i^a K^{[a,b]}_{\phi,ij}(P)\varphi_j^b\right). \qquad (37)$$

The definition of $K^{[a,b]}_{\sigma(\varphi),i}$ is given in Appendix A.

After the renormalization of both the scalar and pseudoscalar fields, analogous to (22), we come to the Lagrangian which can be represented in a form slightly different from that of (37). It is convenient to introduce 4-vectors of “bare” fields

$$\Phi = (\varphi_1^{8\,r}, \varphi_2^{8\,r}, \varphi_1^{9\,r}, \varphi_2^{9\,r}), \qquad \Sigma = (\sigma_1^{8\,r}, \sigma_2^{8\,r}, \sigma_1^{9\,r}, \sigma_2^{9\,r}). \qquad (38)$$

Thus, we have

$$\mathcal{L}^{(2)}(\Sigma,\Phi) = \frac{1}{2}\sum_{i,j=1}^{4}\left(\Sigma_i \mathcal{K}_{\Sigma,ij}(P)\Sigma_j + \Phi_i \mathcal{K}_{\Phi,ij}(P)\Phi_j\right) \qquad (39)$$

where we introduced new functions $\mathcal{K}_{\Sigma(\Phi),ij}(P)$ (see Appendix A).

Up to this moment one has four pseudoscalar and four scalar meson states which are the octet and nonet singlets. The mesons of the same parity have the same quantum numbers and, therefore, they are expected to be mixed. In our model the mixing is represented by 4×4 matrices $R^{\sigma(\varphi)}$ which transform the “bare” fields $\varphi_i^{8\,r}$, $\varphi_i^{9\,r}$, $\sigma_i^{8\,r}$ and $\sigma_i^{9\,r}$ entering the 4-vectors Φ and Σ to the “physical” ones η, η', $\hat\eta$, $\hat\eta'$, σ, $\hat\sigma$, f_0 and $\hat f_0$ represented as components of vectors $\Phi_{\rm ph}$ and $\Sigma_{\rm ph}$:

$$\Phi_{\rm ph} = (\eta, \hat\eta, \eta', \hat\eta'), \qquad \Sigma_{\rm ph} = (\sigma, \hat\sigma, f_0, \hat f_0) \qquad (40)$$

Table 2: The mixing coefficients for the isoscalar meson states

	η	$\hat{\eta}$	η'	$\hat{\eta}'$
φ_1^8	0.71	0.62	-0.32	0.56
φ_2^8	0.11	-0.87	-0.48	-0.54
φ_1^9	0.62	0.19	0.56	-0.67
φ_2^9	0.06	-0.66	0.30	0.82

	σ	$\hat{\sigma}$	f_0	$\hat{f}_0$
σ_1^8	-0.98	-0.66	0.10	0.17
σ_2^8	0.02	1.15	0.26	-0.17
σ_1^9	0.27	-0.09	0.82	0.71
σ_2^9	-0.03	-0.21	0.22	-1.08

where, let us remind once more, a caret over a meson field stands for the first radial excitation of the meson. The transformation $R^{\sigma(\varphi)}$ is linear and nonorthogonal:

$$\Phi_{\text{ph}} = R^{\varphi}\Phi, \qquad \Sigma_{\text{ph}} = R^{\sigma}\Sigma. \tag{41}$$

In terms of "physical" fields the free part of the effective Lagrangian is of the conventional form and the coefficients of matrices $R^{\sigma(\varphi)}$ give the mixing of the $\bar{u}u$ and $\bar{s}s$ components, with and without form factors.

Because of the complexity of the procedure of diagonalization for the matrices of dimensions greater than 2, there is no such simple formulae as, *e.g.*, in (26). Hence, we do not implement it analytically but use numerical methods to obtain matrix elements (see Table 2).

5. Model parameters and meson masses

In our model we have five basic parameters: the masses of the constituent $u(d)$ and s quarks, $m_u = m_d$ and m_s, the cut-off parameter Λ_3, the four-quark coupling constant G and the 't Hooft coupling constant K. We have fixed these parameters with the help of input parameters: the pion decay constant $F_\pi = 93$ MeV, the ρ-meson decay constant $g_\rho = 6.14$ (decay $\rho \to 2\pi$)[6], the masses of pion and kaon and the mass difference of η and η' mesons (for details of these calculations, see [9, 10, 12]). Here we give only numerical estimates of these parameters:

$$\begin{aligned} &m_u = 280 \text{ MeV}, \quad m_s = 405 \text{ MeV}, \quad \Lambda_3 = 1.03 \text{ GeV}, \\ &G = 3.14 \text{ GeV}^{-2}, \quad K = 6.1 \text{ GeV}^{-5}. \end{aligned} \tag{42}$$

[6]Here we do not consider vector and axial-vector mesons, however, we have used the relation $g_\rho = \sqrt{6}g_\sigma$ together with the Goldberger–Treiman relation $g_\pi = \frac{m}{F_\pi} = Z^{-1/2}g_\sigma$ to fix the parameters m_u and Λ_3 (see [9]).

Table 3: The model masses of mesons, MeV

	GR	EXC	$GR(Exp.)$	$EXC(Exp.)$
M_σ	530	1330	$400-1200$	$1200-1500$
M_{f_0}	1070	1600	980 ± 10	1712 ± 5
M_{a_0}	830	1500	983.4 ± 0.9	1474 ± 19
M_π	140	1300	139.56995 ± 0.00035	1300 ± 100
M_K	490	1300	497.672 ± 0.031	1460(?)
$M_{K_0^*}$	960	1500	$-$	1429 ± 12
M_η	520	1280	547.30 ± 0.12	1297.8 ± 2.8
$M_{\eta'}$	910	1470	957.78 ± 0.14	$1440-1470$

We also have a set of additional parameters $c_{qq}^{\sigma^a(\varphi^a)}$ in form factors f_2^a. These parameters are defined by masses of excited pseudoscalar mesons, $c_{uu}^{\pi;a_0} = 1.44$, $c_{uu}^{\eta,\eta',\sigma,f_0} = 1.5$, $c_{us}^{K,K_0^*} = 1.59$, $c_{ss}^{\eta,\eta',\sigma,f_0} = 1.66$. The slope parameters d_{qq} are fixed by special conditions satisfying the standard gap equation, $d_{uu} = -1.78$ GeV^{-2}, $d_{us} = -1.76$ GeV^{-2}, $d_{ss} = -1.73$ GeV^{-2} (see [9]). Using these parmeters, we obtain masses of pseudoscalar and scalar mesons which are listed in Table 3 together with experimental values.

From our calculations we come to the following interpretation of $f_0(1370)$, $f_J(1710)$ and $a_0(1470)$ mesons: we consider them as the first radial excitations of the ground states $f_0(400-1200)$, $f_0(980)$ and $a_0(980)$. Meanwhile, the meson $f_0(1500)$ is much likely a glueball. The strong decays which we consider in the next section substantiate our point of view.

6. Strong decays of the scalar mesons

The ground and excited states of scalar mesons f_0, a_0 decay mostly into pairs of pseudoscalar mesons. In the framework of a quark model and in the leading order of $1/N_c$ expansion, the processes are described by triangle quark diagrams (see Fig.1). Before we start to calculate the amplitudes, corresponding to these diagrams, we introduce, for convenience, Yukawa coupling constants which naturally appear after the renormalization (22) of meson fields:

$$\begin{aligned}
&g_{\sigma_u} \equiv g_{\sigma^a}|_{a=1,2,3,8} = [4I_2(m_u)]^{-1/2}, \quad g_{K_0^*} \equiv g_{\sigma^a}|_{a=4,5,6,7} = [4I_2(m_u,m_s)]^{-1/2}, \\
&g_{\sigma_s} \equiv g_{\sigma^9} = [4I_2(m_s)]^{-1/2}, \quad g_{\varphi^a} = Z^{-1/2} g_{\sigma^a} \\
&g_\pi \equiv g_{\varphi^a}|_{a=1,2,3}, \quad g_K \equiv g_{\varphi^a}|_{a=4,5,6,7}, \quad g_{\varphi_u} \equiv g_{\varphi^8}, \quad g_{\varphi_s} \equiv g_{\varphi^9} \qquad (43)
\end{aligned}$$

$$\begin{aligned}
&\hat{g}_{\sigma_u} \equiv \hat{g}_{\sigma^a}|_{a=1,2,3,8} = [4I_2^{ff}(m_u)]^{-1/2}, \quad \hat{g}_{K_0^*} \equiv \hat{g}_{\sigma^a}|_{a=4,5,6,7} = [4I_2^{ff}(m_u,m_s)]^{-1/2}, \\
&\hat{g}_{\sigma_s} \equiv \hat{g}_{\sigma^9} = [4I_2^{ff}(m_s)]^{-1/2}, \quad \hat{g}_{\varphi^a} = \hat{g}_{\sigma^a}
\end{aligned}$$

$$\hat{g}_\pi \equiv \hat{g}_{\varphi^a}|_{a=1,2,3}\,, \;\; \hat{g}_K \equiv \hat{g}_{\varphi^a}|_{a=4,5,6,7}\,, \;\; \hat{g}_{\varphi_u} \equiv \hat{g}_{\varphi^8}, \;\; \hat{g}_{\varphi_s} \equiv \hat{g}_{\varphi^9} \tag{44}$$

They can easily be related to $Z^a_{\sigma(\varphi),i}$ introduced in the beginning of our paper. Thus, the one-loop contribution to the effective Lagrangian can be rewritten in terms of the renormalized fields:

$$\begin{aligned}\mathcal{L}_{1-\text{loop}}(\sigma,\varphi) &= iN_c \text{Tr}\ln\Big[1 + \frac{1}{i\partial\!\!\!/ - m}\sum_{a=1}^{9}\tau_a\,[g_{\sigma^a}\sigma_1^a + i\gamma_5 g_{\varphi^a}\varphi_1^a + \\ &\qquad (\hat{g}_{\sigma^a}\sigma_2^a + i\gamma_5\hat{g}_{\varphi^a}\varphi_2^a)f_a]\Big]\end{aligned} \tag{45}$$

All amplitudes that describe processes of the type $\sigma \to \varphi_1\varphi_2$ can be divided into two parts:

$$\begin{aligned}T_{\sigma\to\varphi_1\varphi_2} &= C\left(-\frac{iN_c}{(2\pi)^4}\right)\int_{\Lambda_3} d^4k\frac{\text{Tr}[(m+k\!\!\!/+p\!\!\!/_1)\gamma_5(m+k\!\!\!/)\gamma_5(m+k\!\!\!/-p\!\!\!/_2)]}{(m^2-k^2)(m^2-(k+p_1)^2)(m^2-(k-p_2)^2)} \\ &= 4mC\left(-\frac{iN_c}{(2\pi)^4}\right)\int_{\Lambda_3} d^4k\frac{\left[1-\frac{p_1\cdot p_2}{m^2-k^2}\right]}{(m^2-(k+p_1)^2)(m^2-(k-p_2)^2)} \\ &= 4mC[I_2(m,p_1,p_2) - p_1\cdot p_2 I_3(m,p_1,p_2)] = T^{(1)} + T^{(2)}\end{aligned} \tag{46}$$

here $C = 4g_\sigma g_{\varphi_1} g_{\varphi_2}$ and p_1, p_2 are momenta of the pseudoscalar mesons. Using (43) and (44), we rewrite the amplitude $T_{\sigma\to\varphi_1\varphi_2}$ in another form

$$T_{\sigma\to\varphi_1\varphi_2} \approx 4mZ^{-1/2}g_{\varphi_1}\left[1 - p_1\cdot p_2\frac{I_3(m)}{I_2(m)}\right], \tag{47}$$

$$p_1\cdot p_2 = \frac{1}{2}(M_\sigma^2 - M_{\varphi_1}^2 - M_{\varphi_2}^2). \tag{48}$$

We assumed here that the ratio of I_3 to I_2 slowly changes with momentum in comparison with factor $p_1\cdot p_2$, therefore, we ignore their momentum dependence in (47). With this assumption we are going to obtain just a qualitative picture for decays of the excited scalar mesons.

In eqs. (46) and (47) we omitted the contributions from the diagrams which include form factors in vertices. The whole set of diagrams consists of those containing zero, one, two and three form factors. To obtain the complete amplitude, one must sum up all contributions.

After these general comments, let us consider the decays of $a_0(1450)$, $f_0(1370)$ and $f_J(1710)$. First, we estimate the decay width of the process $\hat{a}_0 \to \eta\pi$, taking the mixing coefficients from Table 1 and 2 (see Appendix B for the details). The result is

$$T^{(1)}_{\hat{a}_0\to\eta\pi} \approx 0.2 \text{ GeV}, \tag{49}$$

$$T^{(2)}_{\hat{a}_0\to\eta\pi} \approx 3.5 \text{ GeV}, \tag{50}$$

$$\Gamma_{\hat{a}_0\to\eta\pi} \approx 160 \text{ MeV}. \tag{51}$$

From this calculation one can see that $T^{(1)} \ll T^{(2)}$ and the amplitude is dominated by its second part, $T^{(2)}$, which is momentum dependent. The first part is small because the diagrams with different numbers of form factors cancel each other. As a consequence, in all processes where an excited scalar meson decays into a pair of ground pseudoscalar states, the second part of the amplitude defines the rate of the process.

For the decay $\hat{a}_0 \to \pi\eta'$ we obtain the amplitudes

$$T^{(1)}_{\hat{a}_0\to\pi\eta'} \approx 0.8\ \text{GeV}, \tag{52}$$

$$T^{(2)}_{\hat{a}_0\to\pi\eta'} \approx 3\ \text{GeV}, \tag{53}$$

and the decay width

$$\Gamma_{\hat{a}_0\to\pi\eta'} \approx 36\ \text{MeV}. \tag{54}$$

The decay of $\hat{a}_0$ into kaons is described by the amplitudes $T_{\hat{a}_0\to K^+K^-}$ and $T_{\hat{a}_0\to\bar{K}^0K^0}$ which, in accordance with our scheme, can again be divided into two parts: $T^{(1)}$ and $T^{(2)}$ (see Appendix B for details):

$$T^{(1)}_{\hat{a}_0\to K^+K^-} \approx 0.2\ \text{GeV}, \tag{55}$$

$$T^{(2)}_{\hat{a}_0\to K^+K^-} \approx 2.1\ \text{GeV}. \tag{56}$$

and the decay width is

$$\Gamma_{\hat{a}_0\to KK} = \Gamma_{\hat{a}_0\to K^+K^-} + \Gamma_{\hat{a}_0\to\bar{K}^0K^0} \approx 100\ \text{MeV}. \tag{57}$$

Qualitatively, our results do not contradict the experimental data.

$$\Gamma^{\text{tot}}_{\hat{a}_0} = 265 \pm 13\ \text{MeV}, \quad BR(\hat{a}_0 \to KK) : BR(\hat{a}_0 \to \pi\eta) = 0.88 \pm 0.23. \tag{58}$$

The decay widths of radial excitations of scalar isoscalar mesons are estimated in the same way as it was shown above. We obtain:

$$\Gamma_{\hat{\sigma}\to\pi\pi} = \begin{cases} 550\ \text{MeV}(M_{\hat{\sigma}} = 1.3\ \text{GeV}) \\ 460\ \text{MeV}(M_{\hat{\sigma}} = 1.25\ \text{GeV}), \end{cases} \tag{59}$$

$$\Gamma_{\hat{\sigma}\to\eta\eta} = \begin{cases} 24\ \text{MeV}(M_{\sigma} = 1.3\ \text{GeV}) \\ 15\ \text{MeV}(M_{\sigma} = 1.25\ \text{GeV}), \end{cases} \tag{60}$$

$$\Gamma_{\hat{\sigma}\to\sigma\sigma} = \begin{cases} 6\ \text{MeV}(M_{\sigma} = 1.3\ \text{GeV}) \\ 5\ \text{MeV}(M_{\sigma} = 1.25\ \text{GeV}), \end{cases} \tag{61}$$

$$\Gamma_{\hat{\sigma}\to KK} \sim 5\ \text{MeV}, \tag{62}$$

$$\begin{array}{lcl lcl} \Gamma_{f_0(1710)\to 2\pi} &=& 3\ \text{MeV}, & \Gamma_{f_0(1500)\to 2\pi} &=& 3\ \text{MeV}, \\ \Gamma_{f_0(1710)\to 2\eta} &=& 40\ \text{MeV}, & \Gamma_{f_0(1500)\to 2\eta} &=& 20\ \text{MeV}, \\ \Gamma_{f_0(1710)\to \eta\eta'} &=& 42\ \text{MeV}, & \Gamma_{f_0(1500)\to \eta\eta'} &=& 10\ \text{MeV}, \\ \Gamma_{f_0(1710)\to KK} &=& 24\ \text{MeV}, & \Gamma_{f_0(1500)\to KK} &=& 20\ \text{MeV}. \end{array} \tag{63}$$

The decays of $f_0(1500)$ and $f_0(1710)$ to $\sigma\sigma$ are negligibly small, so we disregard them.

Here we desplayed our estimates for both $f_J(1710)$ and $f_0(1500)$ resonances. Comparing them will allow us to decide which one to consider as the first radial excitation of $f_0(980)$ and which a glueball. From the experimental data:

$$\Gamma^{\mathrm{tot}}_{\sigma'} = 200{-}500\ \mathrm{MeV}, \quad \Gamma^{\mathrm{tot}}_{f_0(1710)} = 133{\pm}14\ \mathrm{MeV}, \quad \Gamma^{\mathrm{tot}}_{f_0(1500)} = 112{\pm}10\ \mathrm{MeV} \tag{64}$$

we can see that in the case of $f_0(1500)$ being a $\bar{q}q$ state there is deficit in the decay widths whereas for $f_J(1710)$ the result is close to experiment. From this we conclude that the meson $f_J(1710)$ is a radially excited partner for $f_0(980)$ and the meson state $f_0(1370)$ is the first radial excitation of $f_0(400-1200)$. As to the state $f_0(1500)$, it is mostly a glueball which significantly contributes to the decay width.

For the decay widths of ground scalar states the situation is opposite to that for excited states. Indeed, the parts $T^{(1)}$ and $T^{(2)}$ of the amplitude T are of the same order of magnitude and different in sign, thus, cancelling each other. The values for decay widths of processes $\sigma \to \pi\pi$, $f_0(980) \to \pi\pi$ and $a_0(980) \to \pi\eta$ turn out to be too small in our approximation:

$$\Gamma_{\sigma\to\pi\pi} \sim 350\ \mathrm{MeV}, \quad \Gamma_{f_0(980)\to\pi\pi} \sim 25\ \mathrm{MeV}, \quad \Gamma_{a_0(980)\to\pi\eta} \sim 1\ \mathrm{MeV}, \tag{65}$$

whereas the experiment gives us

$$\begin{aligned} &\Gamma_{\sigma\to\pi\pi} \sim 600-1000\ \mathrm{MeV}, \quad \Gamma_{f_0(980)\to\pi\pi} \sim 40-100\ \mathrm{MeV}, \\ &\Gamma_{a_0(980)\to\pi\eta} \sim 50-100\ \mathrm{MeV}. \end{aligned} \tag{66}$$

Therefore, to describe correctly strong decays of ground states of scalar mesons, it is necessary to calculate the parts $T^{(1)}$ of the amplitudes more accurately, taking into account their momentum dependence. Indeed, from our previous paper [13] we know that taking into account the quark confinement and the momentum dependence of $I_2(m, p_1, p_2)$ one can find that σ decays into $\pi\pi$ with the width

$$\Gamma_{\sigma\to\pi\pi} \approx 700\ \mathrm{MeV}. \tag{67}$$

For the decay $a_0 \to \eta\pi$, the result can increase more than by an order. A proper description of the decay $a_0(980) \to \pi\eta$ can possibly be obtained from the four-quark interpretation of $a_0(980)$ state [5], and the four-quark component may dominate the process; however, this is beyond our model. For a more careful description of the decay $f_0(980) \to \pi\pi$ one should take into account also the mixing with the glueball state $f_0(1500)$, which we are going to do in our further work.

7. Discussion and Conclusion

Our calculations have shown that we can interpret the scalar states $f_0(1370)$, $a_0(1450)$ and $f_0(1710)$ as the first radial excitations of $f_0(400-1200)$, $a_0(980)$ and $f_0(980)$.

We estimated their masses and the widths of main decays in the framework of a nonlocal chiral quark model. We would like to emphasize that we have not used additional parameters except those necessary to fix the mass spectrum of pseudoscalar mesons. We used the same form factors both for the scalar and pseudoscalar mesons, which is a requirement of the global chiral symmetry.

We assumed that the state $f_0(1500)$ is a glueball, and its probable mixing with $f_0(980)$, $f_0(1370)$ and $f_J(1710)$ may provide us with a more correct description of the masses of these states[7](see Table 3). We are going to consider this problem in a subsequent publication.

More complicated situation takes place for the ground state $a_0(980)$. In the framework of our quark-antiquark model, we have a mass deficit for this meson, 830 MeV instead of 980 MeV and a small decay width for $a_0 \to \pi\eta$. We suspect that this drawback is caused by four-quark component in this state which we did not take into account [5].

There is also some mystery about the strange scalar meson K_0^*. Its experimental mass is large enough, $M_{K_0^*} = 1430$ MeV and the width is 287 ± 23 MeV. In our model, there are two strange scalars, $K_0^*(930)$ with a large width and $K_0^*(1500)$ with the width of the decay $\Gamma_{K_0^* \to K\pi} \sim 300$ MeV. Thus, together with [7] we suppose it is possible for a wide strange resonance, $K_0^*(930)$ to exist in nature still missed in detectors as the ground state whereas the resonance $K_0^*(1430)$ is its radial excitation mostly decaying into $K\pi$. Our model gives for the excited meson K_0^*: $M_{\hat{K}_0^*} \approx 1500$ MeV and $\Gamma_{\hat{K}_0^* \to K\pi} \approx 300$ MeV.

In future we are going to take into account the presence of glueball states and to develop a model with quark confinement for the description of heavy mesons. We will also consider the decays of the excited η and η' mesons.

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Appendix

[7] Our estimates for the masses of f_0 and $\hat{f}_0$: $M_{f_0} = 1070$ MeV and $M_{\hat{f}_0} = 1600$ MeV are expected to shift to $M_{f_0} = 980$ MeV and $M_{\hat{f}_0} = 1710$ MeV after mixing with the glueball $f_0(1500)$.

A Coefficients of the free part of effective Lagrangian for the scalar isoscalar mesons.

The functions $K^{[a,a]}_{\sigma(\varphi),ij}$ introduced in Sec. 4 (37) are defined as follows

$$
\begin{aligned}
K^{[a,a]}_{\sigma(\varphi),11}(P) &= Z^a_{\sigma(\varphi),1}(P^2-(m^a_q\pm m^a_{q'})^2-M^2_{\sigma^a(\varphi^a),1}),\\
K^{[a,a]}_{\sigma(\varphi),22}(P) &= Z^a_{\sigma(\varphi),2}(P^2-(m^a_q\pm m^a_{q'})^2-M^2_{\sigma^a(\varphi^a),2}),\\
K^{[a,a]}_{\sigma(\varphi),12}(P) &= K^{[a,a]}_{\sigma(\varphi),21}(P) \;=\; \gamma^a_{\sigma(\varphi)}(P^2-(m^a_q\pm m^a_{q'})^2),\\
K^{[8,9]}_{\sigma(\varphi),11}(P) &= K^{[9,8]}_{\sigma(\varphi),11}(P)=\frac{1}{2}\left(T^{S(P)}\right)^{-1}_{89},\\
K^{[8,9]}_{\sigma(\varphi),12}(P) &= K^{[9,8]}_{\sigma(\varphi),12}(P)=K^{[8,9]}_{\sigma(\varphi),21}(P)=0,\\
K^{[9,8]}_{\sigma(\varphi),21}(P) &= K^{[8,9]}_{\sigma(\varphi),22}(P)=K^{[9,8]}_{\sigma(\varphi),22}(P)=0.
\end{aligned}
\tag{68}
$$

where the "bare" meson masses are

$$
\begin{aligned}
M^2_{\sigma^8(\varphi^8),1} &= (Z^8_{\sigma(\varphi),1})^{-1}\left(\frac{1}{2}(T^{S(P)})^{-1}_{88}-8I_1(m_u)\right),\\
M^2_{\sigma^9(\varphi^9),1} &= (Z^9_{\sigma(\varphi),1})^{-1}\left(\frac{1}{2}(T^{S(P)})^{-1}_{99}-8I_1(m_s)\right),\\
M^2_{\sigma^8(\varphi^8),2} &= (Z^8_{\sigma(\varphi),2})^{-1}\left(\frac{1}{2G}-8I_1^{ff}(m_u)\right),\\
M^2_{\sigma^9(\varphi^9),2} &= (Z^9_{\sigma(\varphi),2})^{-1}\left(\frac{1}{2G}-8I_1^{ff}(m_s)\right).
\end{aligned}
\tag{69}
$$

In the case of isoscalar mesons it is convenient to combine the scalar and pseudoscalar fields into 4-vectors

$$
\Phi=(\varphi_1^{8\,r},\varphi_2^{8\,r},\varphi_1^{9\,r},\varphi_2^{9\,r}),\qquad \Sigma=(\sigma_1^{8\,r},\sigma_2^{8\,r},\sigma_1^{9\,r},\sigma_2^{9\,r}),
\tag{70}
$$

and introduce 4×4 matrix functions $\mathcal{K}_{\sigma(\varphi),ij}$, instead of old $K^{[a,b]}_{\sigma(\varphi),ij}$, where indices i,j run from 1 through 4. This allows us to rewrite the free part of the effective Lagrangian, which then, with the meson fields renormalized, looks as follows

$$
\mathcal{L}^{(2)}(\Sigma,\Phi)=\frac{1}{2}\sum_{i,j=1}^{4}\left(\Sigma_i\mathcal{K}_{\sigma,ij}(P)\Sigma_j+\Phi_i\mathcal{K}_{\varphi,ij}(P)\Phi_j\right).
\tag{71}
$$

and the functions $\mathcal{K}_{\sigma(\varphi),ij}$ are

$$
\begin{aligned}
\mathcal{K}_{\sigma(\varphi),11}(P) &= P^2-(m_u\pm m_u)^2-M^2_{\sigma^8(\varphi^8),1},\\
\mathcal{K}_{\sigma(\varphi),22}(P) &= P^2-(m_u\pm m_u)^2-M^2_{\sigma^8(\varphi^8),2},\\
\mathcal{K}_{\sigma(\varphi),33}(P) &= P^2-(m_s\pm m_s)-M^2_{\sigma^9(\varphi^9),1},\\
\mathcal{K}_{\sigma(\varphi),44}(P) &= P^2-(m_s\pm m_s)^2-M^2_{\sigma^9(\varphi^9),2},\\
\mathcal{K}_{\sigma(\varphi),12}(P) &= \mathcal{K}_{\sigma(\varphi),21}(P)=\Gamma_{\sigma_u(\eta_u)}(P^2-(m_u\pm m_u)),\\
\mathcal{K}_{\sigma(\varphi),34}(P) &= \mathcal{K}_{\sigma(\varphi),43}(P)=\Gamma_{\sigma_s(\eta_s)}(P^2-(m_s\pm m_s)),\\
\mathcal{K}_{\sigma(\varphi),13}(P) &= \mathcal{K}_{\sigma(\varphi),31}(P)=(Z^8_{\sigma(\varphi),1}Z^9_{\sigma(\varphi),2})^{-1/2}(T^{S(P)})^{-1}_{89}.
\end{aligned}
\tag{72}
$$

Now, to transform (71) to conventional form, one should just diagonalize a 4-dimensional matrix, which is better to do numerically.

B The calculation of the amplitudes for the decays of the excited scalar meson $\hat{a}_0$

Here we collect some instructive formulae, which display a part of details of the calculations made in this work. Let us demonstrate how the amplitude of the decay $\hat{a}_0 \to \eta\pi$ is obtained. The mixing coefficients are taken from Table 1. Moreover, the diagrams where pion vertices contain form factors are neglected because, as one can see from Table 1, their contribution is significantly reduced.

$$\begin{aligned} T^{(1)}_{\hat{a}_0\to\eta\pi} &= 4\frac{m_u^2}{F_\pi}\Big\{0.82\cdot 0.71\cdot Z^{-1/2}\frac{I_2(m_u)}{I_2(m_u)} - \\ &\Big(1.17\cdot 0.71\cdot Z^{-1/2} - 0.82\cdot 0.11\Big)\frac{I_2^f(m_u)}{\sqrt{I_2(m_u)I_2^{ff}(m_u)}} - \\ &1.17\cdot 0.11\cdot\frac{I_2^{ff}(m_u)}{I_2^{ff}(m_u)}\Big\} \approx 0.2\ \text{GeV}, \end{aligned} \tag{73}$$

$$\begin{aligned} T^{(2)}_{\hat{a}_0\to\eta\pi} &= 2\frac{m_u^2}{F_\pi}(M^2_{a_0} - M^2_\eta - M^2_\pi)\Big\{0.82\cdot 0.71 Z^{-1/2}\frac{I_3(m_u)}{I_2(m_u)} - \\ &\Big(1.17\cdot 0.71\cdot Z^{-1/2} - 0.82\cdot 0.11\Big)\frac{I_3^f(m_u)}{\sqrt{I_2(m_u)I_2^{ff}(m)}} - \\ &1.17\cdot 0.11\frac{I_3^{ff}(m_u)}{I_2(m_u)}\Big\} \approx 3.5\ \text{GeV}. \end{aligned} \tag{74}$$

The decay width thereby is

$$\Gamma_{\hat{a}_0\to\eta\pi} = \frac{|T_{\hat{a}_0\to\eta\pi}|^2}{16\pi M^3_{\hat{a}_0}}\sqrt{M^4_{\hat{a}_0}+M^4_\eta+M^4_\pi-2(M^2_{\hat{a}_0}M^2_\eta+M^2_{\hat{a}_0}M^2_\pi+M^2_\eta M^2_\pi)} \approx 160\ \text{MeV}. \tag{75}$$

Here $I_2(m_u) = 0.04$, $I_2^f(m_u) = 0.014c$, $I_2^{ff}(m_u) = 0.015c^2$, $I_3(m_u) = 0.11\ \text{GeV}^{-2}$, $I_3^f(m_u) = 0.07c\ \text{GeV}^{-2}$,$I_3^{ff}(m_u) = 0.06c^2\ \text{GeV}^{-2}$ and c is the external form factor parameter factored out and cancelled in the ratios of the integrals.

For the decay into strange mesons we obtain (see Fig.1)

$$\begin{aligned} &T_{\hat{a}_0\to K^+K^-} = C_K\Big(-\frac{iN_c}{16\pi^2}\Big)\int d^4k\frac{\text{Tr}[(m_u+\not k+\not p_1)\gamma_5(m_s+\not k)\gamma_5(m_u+\not k-\not p_2)]}{(m_s^2-k^2)(m_u^2-(\not k-\not p_1)^2)(m_u^2-(\not k-\not p_2)^2)} \approx \\ &2C_K\Big\{(m_s+m_u)I_2(m_u) - \Delta I_2(m_u,m_s) - [m_s(M^2_{\hat{a}_0} - 2M_K^2) - \\ &\quad 2\Delta^3]I_3(m_u,m_s)\Big\}, \end{aligned} \tag{76}$$

where $\Delta = m_s - m_u$ and

$$I_3(m_u, m_s) = -i\frac{N_c}{(2\pi)^4}\int_{\Lambda_3}\frac{d^4k}{(m_u^2 - k^2)^2(m_s^2 - k^2)}. \quad (77)$$

The coefficient C_K absorbs the Yukawa coupling constants and some structure coefficients. The integral $I_2(m_u, m_s)$ is defined by (21). This is only the part of the amplitude without form factors. The complete amplitude of this process is a sum of contributions which contain also the integrals $I_2^{f..f}$ and $I_3^{f..f}$ with form factors. Thus, the amplitude is

$$T_{\hat{a}_0 \to K^+K^-} = T^{(1)} + T^{(2)}, \quad (78)$$

$$T^{(1)} = \frac{m_u + m_s}{2F_K}\{(m_s + m_u)\cdot 0.13 - \Delta \cdot 0.21\} \approx 0.2 \text{ GeV}, \quad (79)$$

$$T^{(2)} = \frac{m_u + m_s}{2F_K}\{[m_s(M_{a_0}^2 - 2M_K^2) - 2\Delta^3]\cdot 1 \text{ GeV}^{-2}\} \approx 2.3 \text{ GeV}, \quad (80)$$

$$F_K = 1.2F_\pi.$$

The decay width therefore is evaluated to be

$$\Gamma_{\hat{a}_0 \to K^+K^-} = \Gamma_{\hat{a}_0 \to \bar{K}^0 K^0} \approx 50 \text{ MeV}. \quad (81)$$

References

[1] N. Törnqvist, *Phys. Rev. Lett.* **49**, 624 (1982); J. Lanik, *Phys. Lett.* **B306**, 139 (1993); V. Dmitrašinović, *Phys. Rev.* **C53**, 1383 (1996).

[2] Review of Particle Physics, *Europ. Phys. J.* **C3**, 1 (1998)

[3] K. Kusaka, M.K. Volkov, W. Weise, *Phys. Lett.* **B302**, 145 (1993); M. Jaminon, B. Van den Bossche, *Nucl. Phys.* **A619**, 285 (1997).

[4] M.K. Volkov, *Ann. Phys.* **157**, 282 (1984); M.K. Volkov, *Sov. J. Part. Nucl.* **17**, 186 (1986).

[5] R.L. Jaffe, *Phys. Rev.* **D15**, 267, 281 (1977). N.N. Achasov, S.A. Devyanin, G.N. Shestakov, *Usp. Fiz. Nauk.* **142**, 361 (1984);

[6] J. Weinstein and N. Isgur, *Phys. Rev. Lett.* **48**, 659 (1982); J. Weinstein and N. Isgur, *Phys. Rev.* **D27**, 588 (1983);

[7] M.Y. Ishida and S. Ishida, *Progr. Teor. Phys.* **98**, 621 (1997); hep-ph/9712231; M.D. Scadron, *Phys. Rev.* **D26**, 239 (1982).

[8] M.K. Volkov, C. Weiss, *Phys. Rev.* **D56**, 221 (1997).

[9] M.K. Volkov, *Phys. At. Nucl.* **60**, 1920 (1997); hep-ph/9612456.

[10] M.K. Volkov, D. Ebert, M. Nagy, *Int. J. Mod. Phys.* **A13**, 5443 (1998); hep-ph/9705334.

[11] H. Vogl, W. Weise, *Progr. Part. Nucl. Phys.* **27**, 195 (1991); S.P. Klevansky, *Rev. Mod. Phys.* **64**, 649 (1992).

[12] M.K. Volkov, M. Nagy, V.L. Yudichev, *Nuovo Cim.* **A112**, (1999)(in press); JINR Preprint E2-98-101, pp. 8; hep-ph/9804347.

[13] D. Blaschke, G. Burau, M.K. Volkov, V.L. Yudichev, Preprint Rostock Univ. MPG-VT-UR 178/98; hep-ph/9812503; M.K. Volkov, V.L. Yudichev, *Phys. At. Nucl.* **62**, No. 10 (1999) (in press).

Figure 1: Diagrams describing decays of $\hat{a}_0$ to pseudoscalars.